Nordic VLSI ASA

Small loop antennas nAN400-03

1. General

For RF designers developing low-power radio devices for short-range applications, antenna design has become an important issue for the total radio system design. Taking the demand for small size and low cost into account in the development of such radio modules, a small-tuned loop antenna on the same printed circuit board as the radio module is a good solution.

An overview of the basics for electrically small loop antennas is presented. The overview is mainly based on reference [1]. An effective shunt-matching technique for loop antennas, the T-match, is also discussed.

Four different loop antennas for 433 MHz have been fabricated, and impedance and gain measurements have been made on these antennas in an antenna laboratory.

2. Loop antenna basics

Electrically small loop antennas are antennas where the circumference is less than about one-tenth of a wavelength [1]. The field pattern of such loop antennas is similar to that of an infinitesimal dipole with a null perpendicular to the plane of the loop and with its maximum along the plane of the loop.

This chapter describes the geometry and the electrical equivalent circuit for a rectangular loop antenna. Physical dimensions for the antenna is used to calculate the components in the antenna equivalent circuit and the antenna efficiency.

The T-matching method is presented in order to match the impedance of the antenna to a transmitter/receiver.

Formulas for range calculation is also presented in order to make the designer of radio modules for short range applications able to calculate either the range for a device or the power needed for a specified range.

2.1. Loop antenna physical parameters

Figure 1 shows the geometry of the rectangular loop antenna.

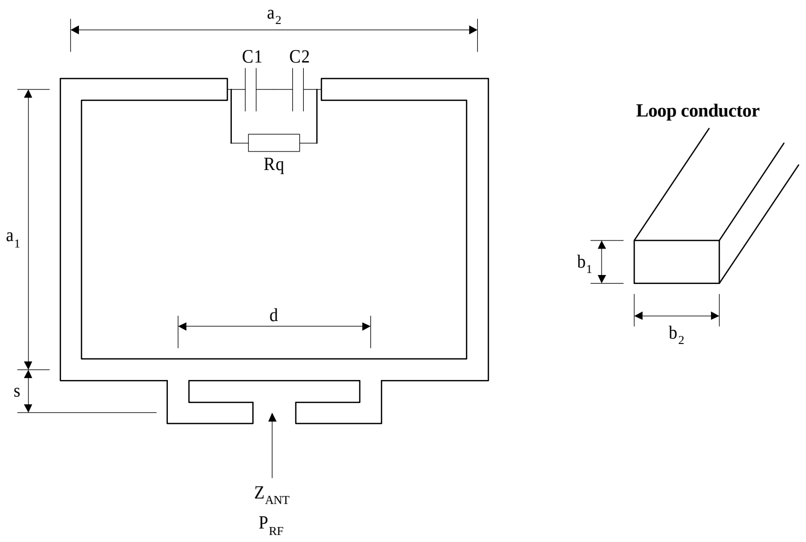

Figure 1. Geometry of rectangular loop antenna

The loop antenna physical parameters used in the calculations are

a_1 = loop antenna width [m]
a_2 = loop antenna length [m]
b_1 = thickness of loop conductor [m]
b_2 = width of loop conductor [m]

For loop antenna fabricated on a printed circuit board (PCB), the thickness of the loop conductor b_1 means the thickness of the copper layer on top of the substrate.

During calculation of the antenna electrical parameters, the rectangular loop has to be modelled as an equivalent quadratic loop, and the planar loop conductor has to be modelled as a wire conductor with equivalent circular radius.

From the parameters above the equivalent quadratic sides of the loop are given by

$$a = \sqrt{a_1 a_2} \quad [m]$$

The calculated equivalent quadratic sides are used in the formulas below for the loop area A, and the inductances L_A and L_I.

The loop area is given by

$$A = a^2 \quad [m^2]$$

The equivalent electrical circular radius of the loop conductor is given by

$$b = 0.35 \cdot b_1 + 0.24 \cdot b_2 \quad [m]$$

In electrostatic, the equivalent radius represents the radius of a circular wire whose capacitance is equal to that of the noncircular geometry, see [1] Table 9.3 pp. 456.

2.2. Loop antenna electrical equivalent circuit

To be able to estimate the capacitor C_P used to resonate the antenna, the input impedance of the loop antenna has to be determined. To estimate the antenna efficiency, radiation resistance, loss resistance of the loop conductor and other ohmic losses has to be determined.

According to [1] the equivalent circuit for the input impedance of a small loop when the loop is used as a transmitting antenna is shown in Figure 2.

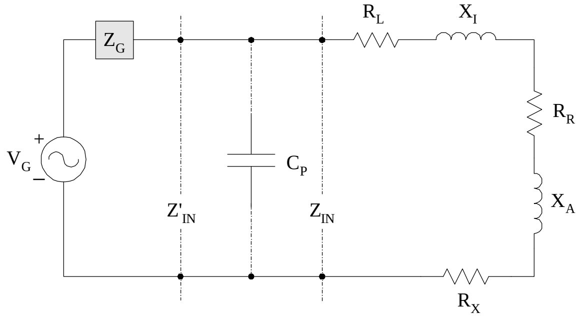

Figure 2. Loop antenna equivalent circuit (transmit mode).

The loop antenna input impedance Z_{IN} is given by:

$$Z_{IN} = (R_R + R_L + R_X) + j2p\, f_0 (L_A + L_I) \quad [\Omega]$$

where

R_R = radiation resistance [ ]
R_L = loss resistance of loop conductor [ ]
Rx = additional ohmic losses (ESR in capacitor C_P etc.) [ ]
L_A = inductance of loop antenna [H]
L_I = inductance of loop conductor [H]

The radiation resistance is given by

$$R_R \approx 31171 \cdot \left(\frac{A^2}{l^4} \right) \quad [\Omega]$$

where

$$l = \frac{c}{f_0} \quad [m]$$

where

c is the speed of light equal to $3 \cdot 10^8$ m/s
f_0 is the resonance frequency in Hz.

The loss resistance of the loop inductor is given by

$$R_L = \frac{l}{P} R_S = \frac{a_1 + a_2}{b_1 + b_2} \sqrt{\frac{p\, f_0 m_0}{s}} \quad [\Omega]$$

where

l = length of the metal loop conductor
P = perimeter of the cross section of the loop conductor
R_S = conductor surface resistance
m_0 = $\cdot 10^{-7}$ H/m
s = conductivity of the conductor equal to $5.8 \cdot 10^7$ S/m for copper.

The additional ohmic losses that is introduced mainly because of ESR (Equivalent Series Resistance) of the capacitor C_P is given by

$$R_X = \frac{2p\, f_0 \left(L_A + L_I\right)}{Q} - R_R - R_L \quad [\Omega]$$

As can be seen from the above expression, the maximum possible quality factor Q of a loop antenna is mainly determined by the ESR (i.e. the quality factor) of the capacitor C_P. A resistor R_Q in parallel with C_P can be used to control the Q-value of the antenna. The insertion of this parallel resistor will reduce the antenna input impedance.

In Figure 2 the capacitor C_P is used in parallel to Z_{IN} to resonate the antenna, that is to cancel out the imaginary part of the input impedance Z_{IN} at the operating frequency. C_P can also be used to represent distributed stray capacitances. It can be shown that the parallel capacitor C_P at resonance is given by

$$C_P = \frac{L_A + L_I}{(R_R + R_L + R_X)^2 + \left[2pf\,(L_A + L_I)\right]^2} \quad [F]$$

Under resonance the input impedance Z'_{IN} can be shown to be equal to

$$Z'_{IN} = R_R + R_L + R_X + \frac{\left[2pf\left(L_A + L_I\right)\right]^2}{R_R + R_L + R_X} \quad [\Omega]$$

The inductive reactance X_A of the loop is computed using the inductance L_A of, [1]

Circular loop of radius a and wire radius b:

$$L_A = m_0 a \left[\ln\left(\frac{8a}{b} \right) - 2 \right] \quad [H]$$

Square loop with sides a and wire radius b:

$$L_A = 2m_0 \frac{a}{p} \left[\ln\left(\frac{a}{b} \right) - 0.774 \right] \quad [H]$$

The reactance X_I of the loop conductor can be computed using the inductance L_I of the loop. For a single turn this can be approximated by [2]

$$L_I = m_0 \frac{A}{2a} \quad [H]$$

where A is the area of the loop.

The antenna efficiency can then be expressed as

$$h = \frac{R_R}{R_R + R_L + R_{ESR}}$$

alternatively

$$h = \frac{QR_R}{2p\, f_0 \left(L_A + L_I \right)}$$

2.3. Impedance matching

Under resonance the resistive input impedance of the loop is high, and has to be transformed down to a lower value to match the transmitter output impedance/receiver input impedance. An effective shunt-matching technique is the T-match connection as shown in Figure 1. This method of matching is based on the fact that the impedance between any two points equidistant from the center along a resonant antenna is resistive, and has a value that depends on the spacing between the two points (feed length). It is therefore possible to choose a pair of points between which the impedance will have the right value to match the transmitter output impedance/receiver input impedance. By reducing the distance between the connection points the impedance is reduced. In practice, the transmitter/receiver cannot be connected directly at these points because the distance between them is much greater than the pin spacing of an integrated circuit. The T-match arrangement in Figure 1 overcomes this difficulty by using a second conductor paralleling the antenna to form a matching section to which the transmitter/receiver may be connected.

A trial and error procedure is used to vary the feed length to make the total input impedance of the loop antenna equal to the transmitter output impedance/receiver input impedance. The estimated capacitor C_P must be tuned for maximum radiated power from the antenna for every position of the connection points.

2.4. Antenna impedance and Q-value with chip capacitors in the loop

The antenna impedance is dependent of both feed length and Q-value (read parallel resistor R_Q). The Q-value is independent of the impedance of the antenna, which means that one chooses a Q-value and then chooses the feed length.

To achieve reproducible values of transmitter power radiation and receiver sensitivity in mass production, the Q-value of the antenna has to match the capacitors that will be used to tune the antenna to the right resonance frequency. The Q-value of the loop should be chosen according to

$$Q = \frac{1}{\sqrt{1 + \frac{tol}{100}} - 1}$$

The *tol* variable is the tolerance of the capacitors in %. The equation is based on the assumption that the variation in radiated power due to capacitor variations should be lower than 3dB. If the tolerance is 4% the Q-value will be 50. If a higher Q-value is chosen, each antenna has to be tuned to keep the variation in radiated power lower than 3dB.

2.5. Range calculations

RF systems operation in the UHF band are not restricted to the line-of-sight coverage of optical systems (IR systems) due to diffraction and reflection of radio waves at edges and conductive surfaces as well as their capability to penetrate dielectric materials.

Range calculation parameters are

- transmitter output power, P_{RF} [dBm]
- transmitter and receiver antenna efficiency, h
- antenna separation, R [m]
- free-space loss, L_P [dB]
- additional propagation losses other than free-space loss, L_X [dB]
- receiver sensitivity, S [dBm]

The free space propagation model is used to predict received signal strength when the transmitter and receiver have a clear, unobstructed line-of-sight path between them. The free-space loss factor L_P is given by

$$L_P = \left(\frac{l}{4p R} \right)^2$$

$$L_P[dB] = 20 \cdot \log\left(\frac{l}{4p R} \right)$$

The free-space loss factor takes into account the losses due to the spherical spreading of the energy by the antenna. The equation shows that the received power will fall off as the square of the transmitter-receiver separation distance. This implies that the received power decays with distance R at a rate of 20 dB/decade, (i.e. 6dB extra loss for doubling of the distance).

Assuming reflection and polarisation-matched antennas, aligned for maximum directional radiation and reception, it can be shown that the communication range with given output power P_{RF}, sensitivity S and equal TX/RX antennas is given by

$$R = \frac{l}{4p \sqrt{\frac{S}{h^2 P_{RF}}}} \quad [m]$$

This equation is based on the assumption that the two antennas are separated by a distance $R > 2D^2 / l$. D is the largest dimension of either antenna. Wave guidance occurring along conductive surfaces may increase the operation range as well.

The free-space path analysis applies to line-of-sight propagation, which means you have to correct for various other propagation losses L_X such as signal reflection, diffraction, scattering and polarisation losses. When these losses are included, the communication range is given by

$$R = \frac{l}{4p \sqrt{\frac{S}{L_X h^2 P_{RF}}}} \quad [m]$$

Given the required range R, assumed losses L_X, sensitivity S and equal TX/RX antennas, the necessary output power P_{RF} is given by

$$P_{RF} = \frac{S}{L_X L_P h^2} \quad [W]$$

2.6. Range calculation example

Given a rectangular loop antenna with dimensions a_1 = 30mm and a_2 = 50mm fabricated on a standard FR-4 substrate. The thickness of the copper layer on top of the substrate is b_1 = 35µm and the width of loop conductor b_2 = 1mm. The antenna quality factor is limited to Q = 50 by a resistor in parallel with the capacitor C_P which is used to resonate the loop. The antenna operates at f_0 = 433.936MHz.

1. Estimate the capacitor C_P at resonance and the antenna efficiency.
2. Calculate the free-space communication range assuming equal loop antennas as given above. The transmitter output power is 10dBm (10mW), and the receiver sensitivity is –103dBm (0.05012 pW).

Solution:

1. To estimate the capacitor C_P, the parameters L_A, L_I, R_R, R_L and R_X has to be calculated. To calculate L_A and L_I, the equivalent quadratic sides, a, and the equivalent electrical circular cylinder radius of the loop conductor b, must be calculated first.

$$a = \sqrt{a_1 a_2} = \sqrt{0.03 \cdot 0.05} = \underline{0.03873\ m}$$

$$b = 0.35 \cdot b1 + 0.24 \cdot b2 = 0.35 \cdot 0.000035 + 0.24 \cdot 0.001 = \underline{0.00025\ m}$$

$$L_A = 2\mu_0 \frac{a}{\pi}\left[\ln\left(\frac{a}{b}\right) - 0.774\right] = 2 \cdot 4\pi \cdot 10^{-7} \cdot \frac{0.03873}{\pi}\left[\ln\left(\frac{0.03873}{0.00025}\right) - 0.774\right]$$

$$L_A = \underline{132.27\ nH}$$

$$L_I = \frac{1}{2}\mu_0 a = \frac{1}{2} \cdot 4\pi \cdot 10^{-7} \cdot 0.03873 = \underline{24.33\ nH}$$

$$R_R \approx 31171 \cdot \left(\frac{A^2}{l^4}\right) = 31171 \cdot \left(\frac{(0.03873 \cdot 0.03873)^2}{(0.6913)^4}\right) = \underline{0.30710\ \Omega}$$

$$R_L = \frac{a_1 + a_2}{b_1 + b_2}\sqrt{\frac{\pi f_0 \mu_0}{\sigma}} = \frac{0.03 + 0.05}{0.000035 + 0.001}\sqrt{\frac{\pi \cdot 433.936 \cdot 10^6 \cdot 4\pi \cdot 10^{-7}}{5.8 \cdot 10^7}} = \underline{0.42008\ \Omega}$$

$$R_X = \frac{2\pi f_0 (L_A + L_I)}{Q} - R_R - R_L$$

$$R_X = \frac{2\pi \cdot 433{,}936 \cdot 10^6 (132.27 \cdot 10^{-9} + 24.33 \cdot 10^{-9})}{50} - 0.30710 - 0.42008$$

$$R_X = \underline{7.81222\ \Omega}$$

Then the capacitor C_P can be calculated:

$$C_P = \frac{L_A + L_I}{(R_R + R_L + R_X)^2 + [2\pi f (L_A + L_I)]^2}$$

$$C_P = \frac{132.27 \cdot 10^{-9} + 24.33 \cdot 10^{-9}}{(0.30710 + 0.42008 + 7.81222)^2 + [2\pi \cdot 433.936 \cdot 10^6 (132.27 \cdot 10^{-9} + 24.33 \cdot 10^{-9})]^2}$$

$$C_P = \underline{\underline{0.86\ pF}}$$

The antenna efficiency is

$$\eta = \frac{R_R}{R_R + R_L + R_{ESR}} = \frac{0.30710}{0.30710 + 0.42008 + 7.81222} = \underline{\underline{0.03596\ (-14.4\ dB)}}$$

2. The communication range R is

$$R = \frac{\lambda}{4\pi \sqrt{\frac{S}{\eta^2 P_{RF}}}} = \frac{0.6913}{4\pi \sqrt{\frac{0.05012 \cdot 10^{-12}}{0.03596^2 \cdot 10 \cdot 10^{-3}}}} = \underline{\underline{884\ m}}$$

3. Loop antenna measurements

Four different tuned loop antennas have been tested in an antenna laboratory. The loop antennas are made on a standard 1.6mm FR4 printed circuit board. The tested loop antenna sizes are:

1. 50x30mm
2. 35x20mm
3. 25x15mm
4. 18x10mm

Each antenna is tuned to a resonance frequency of 433,936MHz with a fixed chip capacitor (5%) in series with a variable capacitor.

All loop antennas are tuned to approximately 400Ω with a T-match. The resistor R_Q for controlling the Q-value is not used in these measurements. The chip capacitor in series with the variable capacitor determines the maximum possible Q-value of the loop antennas. The measured antennas has Q-values of Q = 50±10%.

To be able to compare the measurement results of the loop antennas to some known antenna response, measurements where made on a λ/4 dipole antenna mounted on a 40x40cm ground plane.

We have used a standard log periodic antenna in the antenna laboratory as the transmitter (TX) antenna for all measurements. The λ/4 dipole antenna and the loop antennas under test have been used as receiver (RX) antennas. By doing the measurements this way, we measure the difference between the antennas, not the actual gain for each antenna.

3.1. Antenna transmission

For all measurements, the log periodic dipole is used as the transmitter antenna. The measurement presents S21 for the complete transmission. Both the TX and RX antennas are part of the transmission budget. Due to this we can not extract the absolute gain of the measured λ/4 dipole and loop antennas.

Figure 3 shows the measured S21 for all the loop antennas and the λ/4 dipole antenna.

S21 a = S21 for λ/4 dipole reference antenna
S21 b = S21 for 50x30mm loop antenna
S21 c = S21 for 35x20mm loop antenna
S21 d = S21 for 25x15mm loop antenna
S21 e = S21 for 18x10mm loop antenna

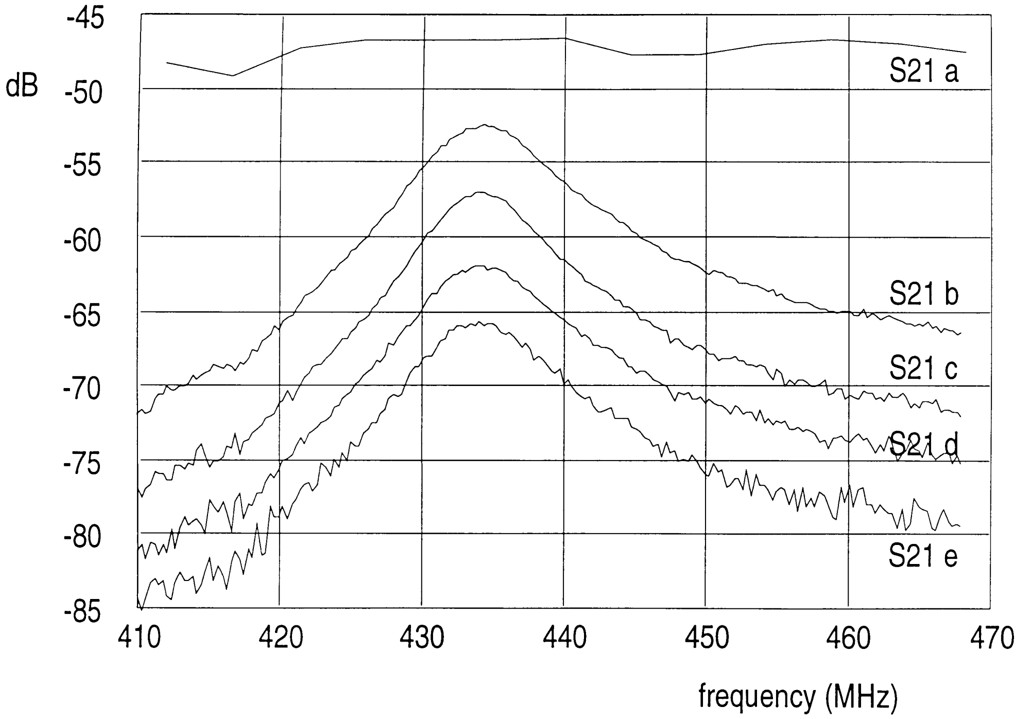

Figure 3. Plot of S21 for all loop antennas and λ/4 dipole reference antenna

Each antenna measurement is discussed in the following chapters.

3.1.1. λ/4 dipole reference antenna

The λ/4 dipole antenna is a whip made of copper and has a length of 16.1cm. The antenna was mounted on a 40x40cm ground plane. From Figure 3 we see that the λ/4 dipole antenna has a measured value of –46.5dB at 434MHz.

3.1.2. 50x30mm loop antenna

This antenna is tuned to 400Ω, and has a quality factor of $Q = 48$. Figure 3 shows a measured peak value of –52,5dB at 434MHz for this antenna. Compared to the λ/4 dipole antenna, the 50x30mm loop antenna has 6dB lower gain.

3.1.3. 35x20mm loop antenna

This antenna is tuned to 386Ω, and has a quality factor of $Q = 54$. Figure 3 shows a measured peak value of –57.5dB at 434MHz for this antenna. Compared to the 50x30mm loop antenna, the 35x20mm loop antenna has 5dB lower gain. The calculated difference in efficiency is 4.1dB. Gerber files for layout is available, see [3, 4, 5].

3.1.4. 25x15mm loop antenna

This antenna is tuned to 414Ω, and has a quality factor of *Q* = 48. Figure 3 shows a measured peak value of –61.5dB at 434MHz for this antenna. Compared to the 50x30mm loop antenna, the 25x15mm loop antenna has 9dB lower gain. The calculated difference in efficiency is 8.4dB. Gerber files for layout is available, see [3, 4, 5].

3.1.5. 18x10mm loop antenna

This antenna is tuned to 400Ω, and has a quality factor of *Q* = 48. Figure 3 shows a measured peak value of –65.5dB at 434MHz for this antenna. Compared to the 50x30mm loop antenna, the 18x10mm loop antenna has 13dB lower gain. The calculated difference in efficiency is 12.8dB. Gerber files for layout is available, see [3, 4, 5].

4. References

1. C. A. Balanis, "*Antenna Theory, Analysis and Design*", second edition, John Wiley & Sons, Inc., 1997.
2. J. D. Kraus, "*Electromagnetics*", 4th ed., McGraw-Hill Book Co., New York, 1992.
3. Application note **nAN400-04**, "nRF0433 RF and antenna layout", Nordic VLSI ASA.
4. Application note **nAN400-05**, "nRF401 RF and antenna layout", Nordic VLSI ASA.
5. Application note **nAN400-06**, "nRF402 RF and antenna layout", Nordic VLSI ASA.

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